Chapter 7

Differential Equations and Mathematical Modeling

Section 7.1 Slope Fields and Euler's Method (pp. 325–335)

Exploration 1 Seeing the Slopes

1. Since $\frac{dy}{dx} = 0$ represents a line with a slope of 0, we should expect to see horizontal slope lines. We see this at odd multiples of $\frac{\pi}{2}$.

2. The formula $\frac{dy}{dx} = \cos x$ depends only on x, not on y.

3. Yes; the curves are vertical translations of each other, so they all have the same slope at any given value of x.

4. At $x = 0$, $\frac{dy}{dx} = \cos 0 = 1$, so the slope at 0 should be 1. That appears to be the slope of each curve as it crosses the y-axis.

5. At $x = \pi$, $\frac{dy}{dx} = \cos \pi = -1$, so the slope should be –1. That appears to be the slope of each curve at $x = \pi$.

6. Yes; the curves themselves are graphs of odd functions, but we see that the *slopes* at the points (x, y) and $(-x, -y)$ are the same.

Quick Review 7.1

1. Yes; $\frac{d}{dx} e^x = e^x$

2. Yes; $\frac{d}{dx} e^{4x} = 4e^{4x}$

3. No; $\frac{d}{dx} (x^2 e^x) = 2xe^x + x^2 e^x$

4. Yes; $\frac{d}{dx} e^{x^2} = 2xe^{x^2}$

5. No; $\frac{d}{dx}(e^{x^2} + 5) = 2xe^{x^2}$

6. Yes; $\frac{d}{dx}\sqrt{2x} = \frac{1}{2\sqrt{2x}}(2) = \frac{1}{\sqrt{2x}}$

7. Yes; $\frac{d}{dx}\sec x = \sec x \tan x$

8. No; $\frac{d}{dx} x^{-1} = -x^{-2}$

9. $y = 3x^2 + 4x + C$
$2 = 3(1)^2 + 4(1) + C$
$C = -5$

10. $y = 2 \sin x - 3 \cos x + C$
$4 = 2 \sin(0) - 3 \cos(0) + C$
$C = 7$

11. $y = e^{2x} + \sec x + C$
$5 = e^{2(0)} + \sec(0) + C$
$C = 3$

12. $y = \tan^{-1} x + \ln(2x - 1) + C$
$\pi = \tan^{-1}(1) + \ln(2(1) - 1) + C$
$C = \frac{3\pi}{4}$

Section 7.1 Exercises

1. $\int dy = \int (5x^4 - \sec^2 x)\, dx$
$y = x^5 - \tan x + C$

2. $\int dy = \int (\sec x \tan x - e^x)\, dx$
$y = \sec x - e^x + C$

3. $\int dy = \int (\sin x - e^{-x} + 8x^3)\, dx$
$y = -\cos x + e^{-x} + 2x^4 + C$

4. $\int dy = \int \left(\frac{1}{x} - \frac{1}{x^2}\right) dx = \ln x + \frac{1}{x} + C$

5. $\int dy = \int \left(5^x \ln 5 + \frac{1}{x^2 + 1}\right) dx = 5^x + \tan^{-1} x + C$

6. $\int dy = \int \left(\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}} \right) dx$
$= \sin^{-1} x - 2\sqrt{x} + C$

7. $\int dy = \int (3t \cos(t^3))\, dt = \sin(t^3) + C$

8. $\int dy = \int \cos t\, e^{\sin t} dt = e^{\sin t} + C$

9. $\int dy = \int (\sec^2(x^5)(5x^4))\, dx = \tan(x^5) + C$

10. $\int dy = \int 4(\sin u)^3 \cos u\, du$
$= (\sin u)^4 + C$
$= \sin^4 u + C$

11. $\int dy = \int 3\sin x\; dx = -3\cos x + C$
$2 = -3\;\cos(0) + C,\; C = 5$
$y = -3\;\cos x + 5$

12. $\int dy = \int 2e^x - \cos x\; dx = 2e^x - \sin x + C$
$3 = 2e^0 - \sin(0) + C,\; C = 1$
$y = 2e^x - \sin x + 1$

13. $\int du = \int (7x^6 - 3x^2 + 5)\, dx = x^7 - x^3 + 5x + C$
$1 = 1^7 - 1^3 + 5 + C,\; C = -4$
$u = x^7 - x^3 + 5x - 4$

14. $\int dA = \int (10x^9 + 5x^4 - 2x + 4)\; dx$
$= x^{10} + x^5 - x^2 + 4x + C$
$6 = 1^{10} + 1^5 - 1^2 + 4(1) + C,\; C = 1$
$A = x^{10} + x^5 - x^2 + 4x + 1$

15. $\int dy = \int \left(-\frac{1}{x^2} - \frac{3}{x^4} + 12 \right) dx$
$= x^{-1} + x^{-3} + 12x + C$
$3 = 1^{-1} + 1^{-3} + 12(1) + C,\; C = -11$
$y = x^{-1} + x^{-3} + 12x - 11 \quad (x > 0)$

16. $\int dy = \int \left(5\sec^2 x - \frac{3}{2}\sqrt{x} \right) dx$
$= 5\tan x - x^{3/2} + C$
$7 = 5\tan(0) - (0)^{3/2} + C,\; C = 7$
$y = 5\tan x - x^{3/2} + 7$

17. $\int dy = \int \left(\frac{1}{1+t^2} + 2^t \ln 2 \right) dt = \tan^{-1} t + 2^t + C$
$3 = \tan^{-1}(0) + 2^0 + C,\; C = 2$
$y = \tan^{-1} t + 2^t + 2$

18. $\int dx = \int \left(\frac{1}{t} - \frac{1}{t^2} + 6 \right) dt = \ln t + t^{-1} + 6t + C$
$0 = \ln(1) + 1^{-1} + 6(1) + C,\; C = -7$
$x = \ln t + t^{-1} + 6t - 7 \qquad (t > 0)$

19. $\int dv = \int (4\sec t\;\tan t + e^t + 6t)\; dt$
$= 4\sec t + e^t + 3t^2 + C$
$5 = 4\;\sec(0) + e^0 + 3(0)^2 + C,\; C = 0$
$v = 4\;\sec t + e^t + 3t^2$
$\left(-\frac{\pi}{2} < t < \frac{\pi}{2} \right)$

20. $\int ds = \int t(3t-2)\; dt = t^3 - t^2 + C$
$0 = (1)^3 - (1)^2 + C,\; C = 0$
$s = t^3 - t^2$

21. $\frac{dy}{dx} = \frac{d}{dx}\int_a^x f(t)\, dt = \frac{d}{dx}\int_1^x \sin(t^2)\, dt$
$y = \int_1^x \sin(t^2)\; dt + 5$

22. $\frac{du}{dx} = \frac{d}{dx}\int_a^x f(t)\, dt = \frac{d}{dx}\int_0^x \sqrt{2+\cos t}\, dt$
$u = \int_0^x \sqrt{2+\cos t}\; dt - 3$

23. $F'(x) = \frac{d}{dx}\int_a^x f(t)\; dt = \frac{d}{dx}\int_2^x e^{\cos t} dt$
$F(x) = \int_2^x e^{\cos t} dt + 9$

24. $G'(s) = \frac{d}{ds}\int_a^s f(t)\, dt = \frac{d}{ds}\int_0^s \sqrt[3]{\tan t}\; dt$
$G(s) = \int_0^s \sqrt[3]{\tan t}\; dt + 4$

25. Graph (b).
$(\sin 0)^2 = 0$
$(\sin 1)^2 > 0$
$(\sin(-1))^2 > 0$

26. Graph (c).

$(\sin 0)^3 = 0$

$(\sin 1)^3 > 0$

$(\sin(-1))^3 < 0$

27. Graph (a).

$(\cos 0)^2 > 0$

$(\cos 1)^2 > 0$

$(\cos(-1))^2 > 0$

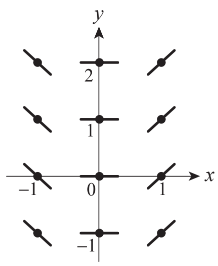
28. Graph (d).

$(\cos 0)^3 > 0$

$(\cos 1)^3 > 0$

$(\cos(-2))^3 < 0$

29.



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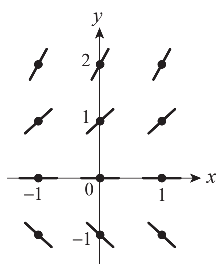

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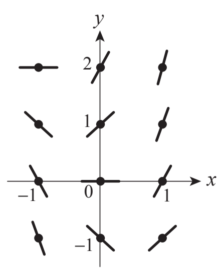

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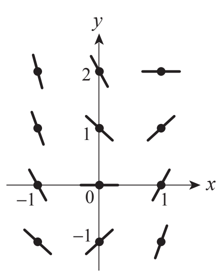

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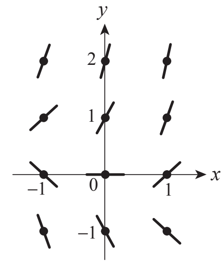

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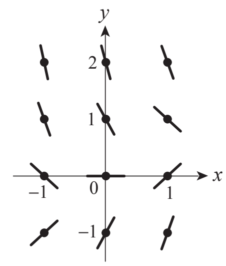
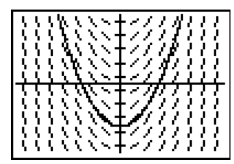
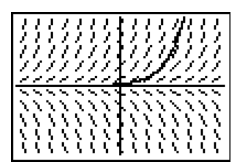


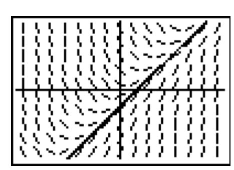
35. When $x = 0$, $\dfrac{dy}{dx} = 0$. The only graph satisfying this condition is graph (c).



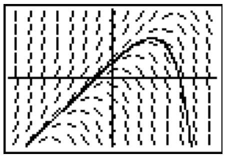
36. When $y > 0$, $\dfrac{dy}{dx} > 0$ and when $y < 0$, $\dfrac{dy}{dx} < 0$. The only graph satisfying these conditions is graph (e).



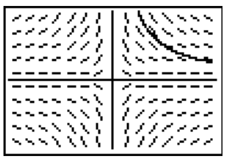
37. When $x > y$, $\dfrac{dy}{dx} > 0$ and when $x < y$, $\dfrac{dy}{dx} < 0$. The only graph satisfying these conditions is graph (a).



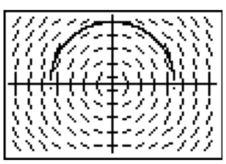
38. When $y > x$, $\frac{dy}{dx} > 0$ and when $y < x$, $\frac{dy}{dx} < 0$. The only graph satisfying these conditions is graph (d).



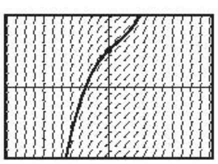
39. In quadrants I and III, $\frac{dy}{dx} < 0$ while in quadrants II and IV, $\frac{dy}{dx} > 0$. When $x = 0$, $\frac{dy}{dx}$ is undefined. The only graph satisfying these conditions is graph (b).



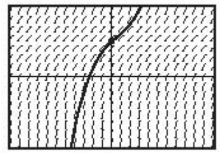
40. In quadrants I and III, $\frac{dy}{dx} < 0$ while in quadrants II and IV, $\frac{dy}{dx} > 0$. When $y = 0$, $\frac{dy}{dx}$ is undefined. The only graph satisfying these conditions is graph (f).



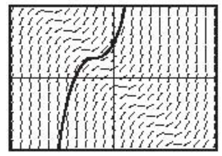
41. When $x = 0$ or $x = 1$, $\frac{dy}{dx} = 1$. The only graph satisfying this condition is graph (d).



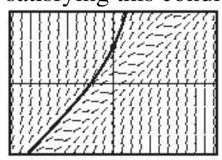
42. When $y = 0$, $\frac{dy}{dx} = \sqrt{5}$. The only graph satisfying this condition is graph (f).



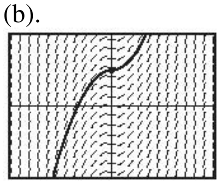
43. When $y = -x$, $\frac{dy}{dx} = 0$. The only graph satisfying this condition is graph (c).



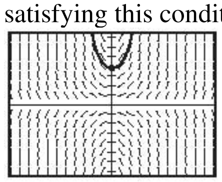
44. When $y = x$, $\frac{dy}{dx} = 0$. The only graph satisfying this condition is graph (e).



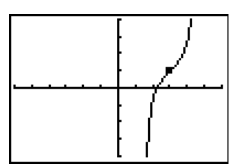
45. When $x = 0$, $\frac{dy}{dx} = 0$. When $x = 1$, $\frac{dy}{dx} = 1$. The only graph satisfying these conditions is graph (b).



46. When $x = 0$ or $y = 0$, $\frac{dy}{dx} = 0$. The only graph satisfying this condition is graph (a).

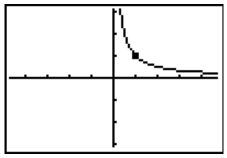


47. **(a)**



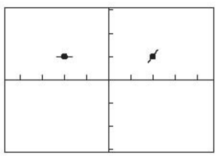
(b) The solution to the initial value problem includes only the continuous portion of the function $y = \tan x + 1$ that passes through the point $(\pi, 1)$.

48. **(a)**

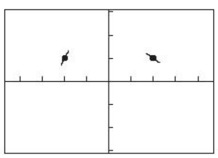


(b) The solution to the initial value problem includes only the continuous portion of the function $y = \frac{1}{x}$ that passes through the point (1, 1).

49. At (–2, 1), $\frac{dy}{dx} = 2(1) + (-2) = 0$ so the correct graph is (c). At (2, 1), $\frac{dy}{dx} = 2(1) + 2 = 4.$



50. At (2, 1), $\frac{dy}{dx} = (1)^2 - 2 = -1$ so the correct graph is (b). At (–2, 1), $\frac{dy}{dx} = (1)^2 - (-2) = 3.$



51.

(x, y)	$\frac{dy}{dx} = x - 1$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 2)	0.0	0.1	0	(1.1, 2)
(1.1, 2)	0.1	0.1	0.01	(1.2, 2.01)
(1.2, 2.01)	0.2	0.1	0.02	(1.3, 2.03)

$y = 2.03$

52.

(x, y)	$\frac{dy}{dx} = y - 1$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 3)	2.0	0.1	0.2	(1.1, 3.2)
(1.1, 3.2)	2.2	0.1	0.22	(1.2, 3.42)
(1.2, 3.42)	2.42	0.1	0.242	(1.3, 3.662)

$y = 3.662$

53.

(x, y)	$\frac{dy}{dx} = y - x$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 2)	1.0	0.1	0.1	(1.1, 2.1)
(1.1, 2.1)	1.0	0.1	0.1	(1.2, 2.2)
(1.2, 2.2)	1.0	0.1	0.1	(1.3, 2.3)

$y = 2.3$

54.

(x, y)	$\frac{dy}{dx} = 2x - y$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 0)	2.0	0.1	0.2	(1.1, 0.2)
(1.1, 0.2)	2.0	0.1	0.2	(1.2, 0.4)
(1.2, 0.4)	2.0	0.1	0.2	(1.3, 0.6)

$y = 0.6$

55.

(x, y)	$\frac{dy}{dx} = 2 - x$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 1)	0.0	–0.1	0.0	(1.9, 1)
(1.9, 1)	0.1	–0.1	–0.01	(1.8, 0.99)
(1.8, 0.99)	0.2	–0.1	–0.02	(1.7, 0.97)

$y = 0.97$

56.

(x, y)	$\frac{dy}{dx} = 1 + y$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 0)	1.0	–0.1	–0.1	(1.9, –0.1)
(1.9, –0.1)	0.9	–0.1	–0.09	(1.8, –0.19)
(1.8, –0.19)	0.81	–0.1	–0.081	(1.7, –0.271)

$y = -0.271$

57.

(x, y)	$\frac{dy}{dx} = x - y$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 2)	–0.0	–0.1	0	(1.9, 2.0)
(1.9, 2)	–0.1	–0.1	0.01	(1.8, 2.01)
(1.8, 2.01)	–0.21	–0.1	0.021	(1.7, 2.031)

$y = 2.031$

58.

(x, y)	$\frac{dy}{dx} = x - 2y$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 1)	0.0	–0.1	0.0	(1.9, 1.0)
(1.9, 1)	–0.1	–0.1	0.01	(1.8, 1.01)
(1.8, 1.01)	–0.22	–0.1	0.022	(1.7, 1.032)

$y = 1.032$

59. **(a)** Graph (b)

(b) The slope is always positive, so (a) and (c) can be ruled out.

60. **(a)** Graph (b)

(b) The solution should have positive slope when x is negative, zero slope when x is zero and negative slope when x is positive since slope $= \frac{dy}{dx} = -x$.
Graphs (a) and (c) don't show this slope pattern.

61. There are positive slopes in the second quadrant of the slope field. The graph of $y = x^2$ has negative slopes in the second quadrant.

62. The slope of $y = \sin x$ would be +1 at the origin, while the slope field shows a slope of zero at every point on the y-axis.

63.

(x, y)	$\frac{dy}{dx} = 2x+1$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 3)	3.0	0.1	0.3	(1.1, 3.3)
(1.1, 3.3)	3.2	0.1	0.32	(1.2, 3.62)
(1.2, 3.62)	3.4	0.1	0.34	(1.3, 3.96)
(1.3, 3.96)	3.6	0.1	0.36	(1.4, 4.32)

$y = 4.32$
Euler's Method gives an estimate $f(1.4) \approx 4.32$.
The solution to the initial value problem is $f(x) = x^2 + x + 1$, from which we get $f(1.4) = 4.36$. The percentage error is thus $\frac{4.36 - 4.32}{4.36} = 0.9\%$.

64.

(x, y)	$\frac{dy}{dx} = 2x-1$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 3)	3.0	–0.1	–0.3	(1.9, 2.7)
(1.9, 2.7)	2.8	–0.1	–0.28	(1.8, 2.42)
(1.8, 2.42)	2.6	–0.1	–0.26	(1.7, 2.16)
(1.7, 2.16)	2.4	–0.1	–0.24	(1.6, 1.92)

$y = 1.92$
Euler's Method gives an estimate $f(1.6) \approx 1.92$. The solution to the initial value problem is $f(x) = x^2 - x + 1$, from which we get $f(1.6) = 1.96$. The percentage error is thus $\frac{1.96 - 1.92}{1.96} = 2\%$.

65. At every (x, y), $(e^{(x-y)/2})(e^{(y-x)/2}) = -e^0 = -1$, so the slopes are negative reciprocals. The slope lines are therefore perpendicular.

66. Since the slopes must be negative reciprocals, $g(x) = -\frac{1}{\sec x} = -\cos x$.

67. The perpendicular slope field would be produced by $\frac{dy}{dx} = -\sin x$, so $y = \cos x + C$ for any constant C.

68. The perpendicular slope field would be produced by $\frac{dy}{dx} = -x$, so $y = -0.5x^2 + C$ for any constant C.

69. True; they are all lines of the form $y = 5x + C$.

70. False; for example, $f(x) = x^2$ is a solution of $\frac{dy}{dx} = 2x$, but $f^{-1}(x) = \sqrt{x}$ is not a solution of $\frac{dy}{dx} = 2y$.

71. C; for all points with $y = 42$, $m = 42 - 42 = 0$

72. E; $y < 0$, $x^2 > 0$, therefore $\frac{dy}{dx} < 0$.

73. B; $y(0) = e^{0^2} = 1$

$$\frac{dy}{dx} = 2xe^{x^2} = 2xy.$$

74. A

75. **(a)** $\frac{dy}{dx} = x - \frac{1}{x^2}$

$$\int \frac{dy}{dx}\,dx = \int (x - x^{-2})\,dx$$

$$y = \frac{x^2}{2} + x^{-1} + C = \frac{x^2}{2} + \frac{1}{x} + C$$

Initial condition: $y(1) = 2$

$$2 = \frac{1^2}{2} + \frac{1}{1} + C$$

$$2 = \frac{3}{2} + C$$

$$\frac{1}{2} = C$$

Solution: $y = \frac{x^2}{2} + \frac{1}{x} + \frac{1}{2},\ x > 0$

(b) Again, $y = \frac{x^2}{2} + \frac{1}{x} + C$.

Initial condition: $y(-1) = 1$

$$1 = \frac{(-1)^2}{2} + \frac{1}{(-1)} + C$$

$$1 = \frac{-1}{2} + C$$

$$\frac{3}{2} = C$$

Solution: $y = \frac{x^2}{2} + \frac{1}{x} + \frac{3}{2},\ x < 0$

(c) For $x < 0$, $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x} + \frac{x^2}{2} + C_1\right)$

$$= -\frac{1}{x^2} + x$$

$$= x - \frac{1}{x^2}.$$

For $x > 0$, $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x} + \frac{x^2}{2} + C_2\right)$

$$= -\frac{1}{x^2} + x$$

$$= x - \frac{1}{x^2}.$$

And for $x = 0$, $\frac{dy}{dx}$ is undefined.

(d) Let C_1 be the value from part (b), and let C_2 be the value from part (a). Thus,

$C_1 = \frac{3}{2}$ and $C_2 = \frac{1}{2}$.

(e) $y(2) = -1$

$$-1 = \frac{1}{2} + \frac{2^2}{2} + C_2$$

$$-1 = \frac{5}{2} + C_2$$

$$-\frac{7}{2} = C_2$$

$y(-2) = 2$

$$2 = \frac{1}{(-2)} + \frac{(-2)^2}{2} + C_1$$

$$2 = \frac{3}{2} + C_1$$

$$\frac{1}{2} = C_1$$

Thus, $C_1 = \frac{1}{2}$ and $C_2 = -\frac{7}{2}$.

76. **(a)** $\frac{d}{dx}(\ln x + C) = \frac{1}{x}$ for $x > 0$

(b) $\frac{d}{dx}[\ln(-x) + C] = \frac{1}{-x}\frac{d}{dx}(-x)$

$$= \left(\frac{1}{-x}\right)(-1)$$

$$= \frac{1}{x}$$

for $x < 0$

(c) For $x > 0$, $\ln|x| + C = \ln x + C$, which is a solution to the differential equation, as we showed in part (a). For $x < 0$, $\ln|x| + C = \ln(-x) + C$, which is a solution to the differential equation, as we showed in part (b). Thus, $\frac{d}{dx}\ln|x| = \frac{1}{x}$ for all x except 0.

(d) For $x < 0$, we have $y = \ln(-x) + C_1$, which is a solution to the differential equation, as we showed in part (a). For $x > 0$, we have $y = \ln x + C_2$, which is a solution to the differential equation, as we showed part (b). Thus, $\frac{dy}{dx} = \frac{1}{x}$ for all x except 0.

77. **(a)**
$$y' = \int (12x + 4)\,dx$$
$$y' = 6x^2 + 4x + C_1$$
$$y = \int (6x^2 + 4x + C_1)\,dx$$
$$y = 2x^3 + 2x^2 + C_1x + C_2$$

(b)
$$y' = \int (e^x + \sin x)\,dx$$
$$y' = e^x - \cos x + C_1$$
$$y = \int (e^x - \cos x + C_1)dx$$
$$y = e^x - \sin x + C_1x + C_2$$

(c)
$$y' = \int (x^3 + x^{-3})\,dx$$
$$y' = \frac{x^4}{4} - \frac{1}{2x^2} + C_1$$
$$y = \int \left(\frac{x^4}{4} - \frac{1}{2x^2} + C_1\right)dx$$
$$y = \frac{x^5}{20} + \frac{1}{2x} + C_1x + C_2$$

78. **(a)**
$$y' = \int (24x^2 - 10)dx$$
$$y' = 8x^3 - 10x + C$$
$$3 = 8(1)^3 - 10(1) + C$$
$$C = 5$$
$$y = \int (8x^3 - 10x + 5)dx$$
$$y = 2x^4 - 5x^2 + 5x + C$$
$$5 = 2(1)^4 - 5(1)^2 + 5(1) + C$$
$$C = 3$$
$$y = 2x^4 - 5x^2 + 5x + 3$$

(b)
$$y' = \int (\cos x - \sin x)dx$$
$$y' = \sin x + \cos x + C$$
$$2 = \sin 0 + \cos 0 + C$$
$$C = 1$$
$$y = \int (\sin x + \cos x + 1)\,dx$$
$$y = -\cos x + \sin x + x + C$$
$$0 = -\cos 0 + \sin 0 + 0 + C$$
$$C = 1$$
$$y = -\cos x + \sin x + x + 1$$

(c)
$$y' = \int (e^x - x)\,dx$$
$$y' = e^x - \frac{x^2}{2} + C$$
$$0 = e^0 - \frac{0^2}{2} + C$$
$$C = -1$$
$$y = \int \left(e^x - \frac{x^2}{2} - 1\right)dx$$
$$y = e^x - \frac{x^3}{6} - x + C$$
$$1 = e^0 - \frac{0^3}{6} - 0 + C$$
$$C = 0$$
$$y = e^x - \frac{x^3}{6} - x$$

79. **(a)**
$$y' = x$$
$$y = \int x\,dx = \frac{x^2}{2} + C$$

(b)
$$y' = -x$$
$$y = \int (-x)\,dx = -\frac{x^2}{2} + C$$

(c)
$$y' = y$$
$$\frac{d}{dx}(Ce^x) = Ce^x$$
$$y = Ce^x$$

(d)
$$y' = -y$$
$$\frac{d}{dx}(Ce^{-x}) = -Ce^{-x}$$
$$y = Ce^{-x}$$

(e) $y' = xy$

$\frac{d}{dx}\left(Ce^{x^2/2}\right) = Cxe^{x^2/2}$

$y = Ce^{x^2/2}$

80. (a) $y'' = x$

$y' = \int x\,dx = \frac{x^2}{2} + C_1$

$y = \int\left(\frac{x^2}{2} + C_1\right)dx = \frac{x^3}{6} + C_1x + C_2$

(b) $y'' = -x$

$y' = \int(-x)\,dx = -\frac{x^2}{2} + C_1$

$y = \int\left(-\frac{x^2}{2} + C_1\right)dx = -\frac{x^3}{6} + C_1x + C_2$

(c) $y'' = -\sin x$

$y' = \int(-\sin x)dx = \cos x + C_1$

$y = \int(\cos x + C_1)\,dx = \sin x + C_1x + C_2$

(d) $y'' = y$

$\frac{d}{dx}(C_1e^x + C_2e^{-x}) = C_1e^x - C_2e^{-x} = y'$

$\frac{d}{dx}(C_1e^x - C_2e^{-x}) = C_1e^x + C_2e^{-x} = y''$

$y = C_1e^x + C_2e^{-x}$

(e) $y'' = -y$

$\frac{d}{dx}(C_1 \sin x + C_2 \cos x)$

$= C_1 \cos x - C_2 \sin x$

$= y'$

$\frac{d}{dx}(C_1 \cos x - C_2 \sin x)$

$= -C_1 \sin x - C_2 \cos x$

$y = C_1 \sin x + C_2 \cos x$

Section 7.2 Antidifferentiation by Substitution (pp. 336–344)

Exploration 1 Are $\int f(u)\,du$ and $\int f(u)\,dx$ the Same Thing?

1. $\int f(u)\,du = \int u^3 du = \frac{u^4}{4} + C$

2. $\frac{u^4}{4} + C = \frac{(x^2)^4}{4} + C = \frac{x^8}{4} + C$

3. $f(u) = u^3 = (x^2)^3 = x^6$

$\int f(u)\,dx = \int x^6 dx = \frac{x^7}{7} + C$

4. No

Quick Review 7.2

1. $\int_0^2 x^4 dx = \frac{1}{5}x^5\Big|_0^2 = \frac{1}{5}(2)^5 - \frac{1}{5}(0)^5 = \frac{32}{5}$

2. $\int_1^5 \sqrt{x-1}\,dx = \int_1^5 (x-1)^{1/2}\,dx$

$= \frac{2}{3}(x-1)^{3/2}\Big|_1^5$

$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$

$= \frac{2}{3}(8) = \frac{16}{3}$

3. $\frac{dy}{dx} = 3^x$

4. $\frac{dy}{dx} = 3^x$

5. $\frac{dy}{dx} = 4(x^3 - 2x^2 + 3)^3(3x^2 - 4x)$

6. $\frac{dy}{dx} = 2\sin(4x-5)\cos(4x-5)\cdot 4$

$= 8\sin(4x-5)\cos(4x-5)$

7. $\frac{dy}{dx} = \frac{1}{\cos x}\cdot -\sin x = -\tan x$

8. $\frac{dy}{dx} = \frac{1}{\sin x}\cdot \cos x = \cot x$

9. $\frac{dy}{dx} = \frac{1}{\sec x + \tan x}\cdot(\sec x \tan x + \sec^2 x)$

$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$

$= \sec x$

10. $\frac{dy}{dx} = \frac{1}{\csc x + \cot x}(-\csc x \cot x - \csc^2 x)$

$= -\frac{\csc x \cot x + \csc^2 x}{\csc x + \cot x}$

$= -\frac{\csc x(\cot x + \csc x)}{\csc x + \cot x}$

$= -\csc x$

Section 7.2 Exercises

1. $\int (\cos x - 3x^2)\, dx = \sin x - x^3 + C$

2. $\int x^{-2} dx = -x^{-1} + C$

3. $\int \left(t^2 - \frac{1}{t^2}\right) dt = \frac{t^3}{3} + t^{-1} + C$

4. $\int \frac{dt}{t^2 + 1} = \tan^{-1} t + C$

5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\, dx$

$= \frac{3}{5}x^5 + x^{-2} + \tan x + C$

6. $\int \left(2e^x + \sec x \tan x - \sqrt{x}\right) dx$

$= 2e^x + \sec x - \frac{2}{3}x^{3/2} + C$

7. $(-\cot u + C)' = -(-\csc^2 u) = \csc^2 u$

8. $(-\csc u + C)' = -(-\csc u \cot u) = \csc u \cot u$

9. $\left(\frac{1}{2}e^{2x} + C\right)' = \frac{1}{2}e^{2x}(2) = e^{2x}$

10. $\left(\frac{1}{\ln 5}5^x + C\right)' = \frac{1}{\ln 5}\, 5^x(\ln 5) = 5^x$

11. $(\tan^{-1} u + C)' = \frac{1}{1 + u^2}$

12. $(\sin^{-1} u + C)' = \frac{1}{\sqrt{1 - u^2}}$

13. $\int f(u)\, du = \int \sqrt{u}\, du$

$= \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}x^3 + C$

$\int f(u)\, dx = \int \sqrt{u}\, dx$

$= \int \sqrt{x^2} dx$

$= \int x\, dx$

$= \frac{1}{2}x^2 + C$

14. $\int f(u)\, du = \int u^2\, du = \frac{1}{3}u^3 + C = \frac{1}{3}x^{15} + C$

$\int f(u)\, dx = \int u^2\, dx = \int x^{10}\, dx = \frac{1}{11}x^{11} + C$

15. $\int f(u)\, du = \int e^u\, du = e^u + C = e^{7x} + C$

$\int f(u)\, du = \int e^u dx = \int e^{7x} dx = \frac{1}{7}e^{7x} + C$

16. $\int f(u)\, du = \int \sin u\, du$

$= -\cos u + C$

$= -\cos 4x + C$

$\int f(u)\, dx = \int \sin u\, dx$

$= \int \sin 4x\, dx$

$= -\frac{1}{4}\cos 4x + C$

17. $u = 3x$

$du = 3\, dx$

$\frac{1}{3}du = dx$

$\int \sin 3x\, dx = \frac{1}{3}\int \sin u\, du$

$= -\frac{1}{3}\cos u + C$

$= -\frac{1}{3}\cos 3x + C$

Check:

$\frac{d}{dx}\left(-\frac{1}{3}\cos 3x + C\right) = -\frac{1}{3}(-\sin 3x)(3) = \sin 3x$

18. $u = 2x^2$

$du = 4x\, dx$

$x\, dx = \frac{1}{4}du$

$$\int x\cos(2x^2)\,dx = \frac{1}{4}\int \cos u\,du$$
$$= \frac{1}{4}\sin u + C$$
$$= \frac{1}{4}\sin(2x^2) + C$$

Check: $\frac{d}{dx}\left(\frac{1}{4}\sin(2x^2)+C\right) = \frac{1}{4}\cos(2x^2)(4x)$
$$= x\cos(2x^2)$$

19. $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int \sec u\tan u\,du$$
$$= \frac{1}{2}\sec u + C$$
$$= \frac{1}{2}\sec 2x + C$$

Check: $\frac{d}{dx}\left(\frac{1}{2}\sec 2x + C\right) = \frac{1}{2}\sec 2x \tan 2x \cdot 2$
$$= \sec 2x \tan 2x$$

20. $u = 7x - 2$

$du = 7dx$

$\frac{1}{7}du = dx$

$$\int 28(7x-2)^3\,dx = \frac{1}{7}\int 28u^3 du$$
$$= u^4 + C$$
$$= (7x-2)^4 + C$$

Check: $\frac{d}{dx}\left[(7x-2)^4 + C\right] = 4(7x-2)^3(7)$
$$= 28(7x-2)^3$$

21. $u = \frac{x}{3}$ $\qquad x = 3u$

$du = \frac{1}{3}dx$ $\qquad x^2 = 9u^2$

$3\,du = dx$

$$\int \frac{dx}{x^2+9} = \int \frac{3du}{9u^2+9}$$
$$= \frac{3}{9}\int \frac{du}{u^2+1}$$
$$= \frac{1}{3}\int \frac{du}{u^2+1}$$
$$= \frac{1}{3}\tan^{-1} u + C$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Check:
$$\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\frac{x}{3} + C\right) = \frac{1}{3}\frac{1}{1+\left(\frac{x}{3}\right)^2}\cdot\frac{1}{3} = \frac{1}{9+x^2}$$

22. $u = 1 - r^3$

$du = -3r^2\,dr$

$-\frac{1}{3}du = r^2 dr$

$$\int \frac{9r^2 dr}{\sqrt{1-r^3}} = 9\left(-\frac{1}{3}\right)\int \frac{du}{\sqrt{u}}$$
$$= -3\int u^{-1/2} du$$
$$= -3(2)u^{1/2} + C$$
$$= -6\sqrt{1-r^3} + C$$

Check:
$$\frac{d}{dx}\left(-6\sqrt{1-r^3}+C\right) = -6\left(\frac{1}{2\sqrt{1-r^3}}\right)(-3r^2)$$
$$= \frac{9r^2}{\sqrt{1-r^3}}$$

23. $u = 1 - \cos\frac{t}{2}$

$du = \frac{1}{2}\sin\frac{t}{2}dt$

$2\,du = \sin\frac{t}{2}dt$

$$\int\left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}dt = 2\int u^2 du$$
$$= \frac{2}{3}u^3 + C$$
$$= \frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C$$

Check: $\frac{d}{dx}\left[\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3+C\right]$
$= 2\left(1-\cos\frac{t}{2}\right)^2\left(\sin\frac{t}{2}\right)\left(\frac{1}{2}\right)$
$= \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}$

24. $u = y^4 + 4y^2 + 1$
$du = (4y^3 + 8y)\ dy$
$du = 4(y^3 + 2y)\ dy$
$\frac{1}{4}du = (y^3 + 2y)\,dy$
$\int 8(y^4+4y^2+1)^2(y^3+2y)dy$
$= 8\left(\frac{1}{4}\right)\int u^2 du$
$= \frac{2}{3}u^3 + C$
$= \frac{2}{3}(y^4+4y^2+1)^3 + C$
Check: $\frac{d}{dx}\left[\frac{2}{3}(y^4+4y^2+1)^3+C\right]$
$= 2(y^4+4y^2+1)^2(4y^3+8y)$
$= 8(y^4+4y^2+1)^2(y^3+2y)$

25. Let $u = 1-x$
$du = -dx$
$\int \frac{dx}{(1-x)^2} = -\int \frac{du}{u^2}$
$= u^{-1} + C$
$= \frac{1}{1-x} + C$

26. Let $u = x+2$
$du = dx$
$\int \sec^2(x+2)\ dx = \int \sec^2 u\ du$
$= \tan u + C$
$= \tan(x+2) + C$

27. Let $u = \tan x$
$du = \sec^2 x\ dx$
$\int \sqrt{\tan x}\sec^2 x\ dx = \int u^{1/2} du$
$= \frac{2}{3}u^{3/2} + C$
$= \frac{2}{3}(\tan x)^{3/2} + C$

28. Let $u = \theta + \frac{\pi}{2}$
$du = d\theta$
$\int \sec\left(\theta+\frac{\pi}{2}\right)\tan\left(\theta+\frac{\pi}{2}\right)d\theta$
$= \int \sec\ u \tan\ u\ du$
$= \sec u + C$
$= \sec\left(\theta+\frac{\pi}{2}\right) + C$

29. $\int \tan(4x+2)\ dx$
$u = 4x+2$
$du = 4\ dx$
$\frac{1}{4}du = dx$
$\frac{1}{4}\int \tan u\ du$
$= -\frac{1}{4}\ln\left|\cos(4x+2)\right| + C$ or
$\frac{1}{4}\ln\left|\sec(4x+2)\right| + C$

30. $\int 3(\sin x)^{-2}dx = 3\int \frac{1}{\sin^2 x}dx$
$= 3\int \csc^2 x\ dx$
$= -3\cot x + C$

31. Let $u = 3z+4$
$du = 3\ dz$
$\frac{1}{3}du = dz$
$\int \cos(3z+4)dz = \frac{1}{3}\int \cos\ u\ du$
$= \frac{1}{3}\sin u + C$
$= \frac{1}{3}\sin(3z+4) + C$

32. Let $u = \cot x$
$du = -\csc^2 x\ dx$
$\int \sqrt{\cot x}\csc^2 x\ dx = -\int u^{1/2}du$
$= -\frac{2}{3}u^{3/2} + C$
$= -\frac{2}{3}(\cot x)^{3/2} + C$

33. Let $u = \ln x$

$$du = \frac{1}{x}dx$$

$$\int \frac{\ln^6 x}{x}\,dx = \int u^6 du$$
$$= \frac{1}{7}u^7 + C$$
$$= \frac{1}{7}(\ln^7 x) + C$$

34. Let $u = \tan\left(\frac{x}{2}\right)$

$$du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$$

$$\int \tan^7\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx = 2\int u^7 du$$
$$= 2 \cdot \frac{1}{8}u^8 + C$$
$$= \frac{1}{4}\tan^8\left(\frac{x}{2}\right) + C$$

35. Let $u = s^{4/3} - 8$

$$du = \frac{4}{3}s^{1/3}ds$$

$$\frac{3}{4}du = s^{1/3}ds$$

$$\int s^{1/3}\cos(s^{4/3} - 8)\,ds = \frac{3}{4}\int \cos u\,du$$
$$= \frac{3}{4}\sin u + C$$
$$= \frac{3}{4}\sin(s^{4/3} - 8) + C$$

36. $\int \frac{dx}{\sin^2 3x} = \int \csc^2 3x\,dx$

Let $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$\int \csc^2 3x\,dx = \frac{1}{3}\int \csc^2 u\,du$$
$$= -\frac{1}{3}\cot u + C$$
$$= -\frac{1}{3}\cot(3x) + C$$

37. Let $u = \cos(2t+1)$

$$du = -\sin(2t+1)(2)dt$$

$$-\frac{1}{2}du = \sin(2t+1)dt$$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt = -\frac{1}{2}\int u^{-2}du$$
$$= \frac{1}{2}u^{-1} + C$$
$$= \frac{1}{2\cos(2t+1)} + C$$
$$= \frac{1}{2}\sec(2t+1) + C$$

38. Let $u = 2 + \sin t$

$du = \cos t\;dt$

$$\int \frac{6\cos t}{(2+\sin t)^2}dt = 6\int u^{-2}du$$
$$= -6u^{-1} + C$$
$$= -\frac{6}{2+\sin t} + C$$

39. $\int \frac{dx}{x\ln x}$

$u = \ln x$

$du = \frac{dx}{x}$

$x\,du = dx$

$\int \frac{du}{u} = \ln\;u = \ln(\ln x) + C$

40. $\int \tan^2 x\sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}\tan^3 x + C$

41. $\int \frac{x\,dx}{x^2+1}$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{du}{x^2+1} = \frac{1}{2}\ln\;u + C = \frac{1}{2}\ln(x^2+1) + C$

42. Let $u = \frac{x}{5}$ $\qquad 5u = x$

$du = \frac{1}{5}dx \quad 25u^2 = x^2$

$5du = dx$

$$\int \frac{40\,dx}{x^2+25} = \int \frac{200\,du}{25u^2+5} = \frac{200}{25}\int \frac{du}{u^2+1} = 8\tan^{-1} u + C = 8\tan^{-1}\left(\frac{x}{5}\right) + C$$

43. $\int \frac{dx}{\cot 3x} = \int \frac{\sin 3x}{\cos 3x}dx$

Let $u = \cos 3x$

$du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin 3x\,dx$

$$\int \frac{dx}{\cot 3x} = -\frac{1}{3}\int \frac{1}{u}du = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|\cos 3x| + C$$

(An equivalent expression is $\frac{1}{3}\ln\ |\sec 3x| + C$.)

44. Let $u = 5x + 8$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{1}{5}\int u^{-1/2}\,du = \frac{1}{5}\cdot 2u^{1/2} + C = \frac{2}{5}\sqrt{5x+8} + C$$

45. $\int \sec x\,dx = \int \sec x \cdot \left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx$

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$

Let $u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x\,dx$

$$\int \sec x\,dx = \int \frac{1}{u}du = \ln|u| + C = \ln|\sec x + \tan x| + C$$

46. $\int \csc x\,dx = \int \csc x\left(\frac{\csc x + \cot x}{\csc x + \cot x}\right)dx$

$= \int \frac{\csc^2 x + \csc x\cot x}{\csc x + \cot x}\,dx$

Let $u = \csc x + \cot x$

$du = -\csc x\cot x - \csc^2 x\,dx$

$$\int \csc x\,dx = -\int \frac{1}{u}du = -\ln|u| + C = -\ln|\csc x + \cot x| + C$$

47. $\int \sin^3 2x\,dx = \int (\sin^2 2x)\cdot \sin 2x\,dx$

$= \int (1 - \cos^2 2x)\cdot \sin 2x\,dx$

Let $u = \cos 2x$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx$

$$= -\frac{1}{2}\int (1-u^2)du = -\frac{1}{2}\left(u - \frac{u^3}{3}\right) + C = -\frac{u}{2} + \frac{u^3}{6} + C = -\frac{\cos 2x}{2} + \frac{\cos^3 2x}{6} + C$$

48. $\int \sec^4 x\,dx = \int (\sec^2 x)\sec^2 x\,dx$

$= \int (1 + \tan^2 x)\sec^2 x\,dx$

Let $u = \tan x$

$du = \sec^2 x\,dx$

$$= \int (1+u^2)du = u + \frac{u^3}{3} + C = \tan x + \frac{\tan^3 x}{3} + C$$

49. $\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\int 2\sin^2 x\,dx = \int (1 - \cos 2x)\,dx$

Let $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= \frac{1}{2}\int (1 - \cos u)\,du$

$= \frac{1}{2}(u - \sin u) + C$

$= \frac{1}{2}(2x - \sin 2x) + C$

$= x - \frac{\sin 2x}{2} + C$

50. $\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\int 4\cos^2 x\,dx = \int 2(1 + \cos 2x)\,dx$

Let $u = 2x$

$du = 2\,dx$

$= \int (1 + \cos u)\,du$

$= u + \sin u + C$

$= 2x + \sin 2x + C$

51. $\int \tan^4 x\,dx$

$= \int \tan^2 x \cdot \tan^2 x\,dx$

$= \int \tan^2 x\,(\sec^2 x - 1)\,dx$

$= \int (\tan^2 x \sec^2 x - \tan^2 x)\,dx$

$= \int (\tan^2 x \sec^2 x - \sec^2 x + 1)\,dx$

$= \int (\tan^2 x \sec^2 x - \sec^2 x)\,dx + \int 1\,dx$

$= \int (\tan^2 x - 1)\sec^2 x\,dx + \int dx$

Let $u = \tan x$

$du = \sec^2 x\,dx$

$= \int (u^2 - 1)\,du + \int dx$

$= \frac{1}{3}u^3 - u + x + C$

$= \frac{1}{3}\tan^3 x - \tan x + x + C$

52. $\int (\cos^4 x - \sin^4 x)\,dx$

$= \int (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)\,dx$

$= \int (1)(\cos 2x)\,dx$

$= \frac{1}{2}\sin 2x + C$

53. Let $u = y + 1$ $\quad u(0) = 0 + 1 = 1$

$du = dy$ $\quad u(3) = 3 + 1 = 4$

$\int_0^3 \sqrt{y+1}\,dy = \int_1^4 u^{1/2}\,du$

$= \frac{2}{3}u^{3/2}\Big|_1^4$

$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$

$= \frac{2}{8}(8) - \frac{2}{3}$

$= \frac{14}{3}$

54. Let $u = 1 - r^2$ $\quad u(0) = 1 - 0^2 = 1$

$du = -2r\,dr$ $\quad u(1) = 1 - 1^2 = 0$

$-\frac{1}{2}du = r\,dr$

$\int_0^1 r\sqrt{1-r^2}\,dr = -\frac{1}{2}\int_1^0 u^{1/2}\,du = -\frac{1}{3}u^{3/2}\Big|_1^0$

$= -\frac{1}{3}0^{3/2} + \frac{1}{3}\cdot 1^{3/2}$

$= \frac{1}{3}$

55. Let $u = \tan x$, $u\left(-\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right) = -1$,

$u(0) = \tan(0) = 0$

$du = \sec^2 x\,dx$

$\int_{-\pi/4}^0 \tan x \sec^2 x\,dx = \int_{-1}^0 u\,du$

$= \frac{1}{2}u^2\Big|_{-1}^0$

$= \frac{1}{2}(0) - \frac{1}{2}(-1)^2$

$= -\frac{1}{2}$

56. Let $u = 4 + r^2$ $\quad u(1) = 4 + 1^2 = 5$

$du = 2r\ dr$ $\quad u(-1) = 4 + (-1)^2 = 5$

$\frac{1}{2} du = r\ dr$

$$\int_{-1}^{1} \frac{5r}{(4+r^2)^2} dr = \frac{5}{2}\int_5^5 u^{-2}\, du = 0$$

57. Let $u = 1 + \theta^{3/2}$ $\quad u(0) = 1 + 0 = 1$

$du = \frac{3}{2}\theta^{1/2}\, d\theta$ $\quad u(0) = 1 + 1 = 2$

$\frac{2}{3} du = \theta^{1/2}\, d\theta$

$$\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2} d\theta = \frac{2}{3}(10)\int_1^2 u^{-2}\, du$$

$$= -\frac{20}{3} u^{-1}\Big|_1^2$$

$$= -\frac{20}{3}\left(\frac{1}{2} - 1\right)$$

$$= -\frac{20}{3}\left(-\frac{1}{2}\right)$$

$$= \frac{10}{3}$$

58. Let $u = 4 + 3 \sin x$

$du = 3 \cos x\, dx$

$u(-\pi) = 4 + 3\sin(-\pi) = 4$

$u(\pi) = 4 + 3\sin \pi = 4$

$\frac{1}{3} du = \cos x\ dx$

$$\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4 + 3 \sin x}} dx = \frac{1}{3}\int_4^4 u^{-1/2}\, du = 0$$

59. Let $u = t^5 + 2t$ $\quad u(0) = 0 + 0 = 0$

$du = (5t^4 + 2)\, dt$ $\quad u(1) = 1 + 2 = 3$

$$\int_0^1 \sqrt{t^5 + 2t}\ (5t^4 + 2)\, dt = \int_0^3 u^{1/2}\, du$$

$$= \frac{2}{3} u^{3/2}\Big|_0^3$$

$$= \frac{2}{3}(3)^{3/2}$$

$$= \frac{2}{3}\sqrt{27}$$

$$= 2\sqrt{3}$$

60. Let $u = \cos 2\theta$ $\quad u(0) = \cos 0 = 1$

$du = -2 \sin 2\theta\ d\theta$ $\quad u\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$-\frac{1}{2} du = \sin 2\theta\ d\theta$

$$\int_0^{\pi/6} \cos^{-3} 2\theta \sin 2\theta\ d\theta$$

$$= -\frac{1}{2}\int_1^{1/2} u^{-3}\, du$$

$$= -\frac{1}{2} \cdot \left(-\frac{1}{2}\right) u^{-2}\Big|_1^{1/2}$$

$$= \frac{1}{4}\left(\left(\frac{1}{2}\right)^{-2} - 1\right)$$

$$= \frac{1}{4}(3)$$

$$= \frac{3}{4}$$

61. $\int_0^7 \frac{dx}{x+2}$

$u = x + 2$ $\quad u(0) = 0 + 2 = 2$

$du = dx$ $\quad u(7) = 7 + 2 = 9$

$$\int_2^9 \frac{du}{u} = \ln|u|\Big|_2^9 = \ln\left(\frac{9}{2}\right) = 1.504$$

62. $\int_2^5 \frac{dx}{2x-3}$

$u = 2x - 3$ $\quad u(2) = 4 - 3 = 1$

$du = 2\, dx$ $\quad u(5) = 10 - 3 = 7$

$\frac{1}{2} du = dx$

$$\frac{1}{2}\int_1^7 \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_1^7 = \frac{1}{2}\ln 7 - \frac{1}{2}\ln 1 = \frac{1}{2}\ln 7$$

63. $\int_1^2 \frac{dt}{t-3}$

$u = t - 3$ $\quad u(1) = 1 - 3 = -2$

$du = dt$ $\quad u(2) = 2 - 3 = -1$

$$\int_{-2}^{-1} \frac{du}{u} = \ln|u|\Big|_{-2}^{-1}$$

$$= \ln|-1| - \ln|-2|$$

$$= \ln 1 - \ln 2$$

$$= \ln\left(\frac{1}{2}\right)$$

64. $\int_{\pi/4}^{3\pi/4} \cot x \; dx = \int_{\pi/4}^{3\pi/4} \frac{\cos x \, dx}{\sin x}$

$u = \sin x \qquad u\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$du = \cos x \, dx \qquad u\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\int_{\sqrt{2}/2}^{\sqrt{2}/2} \frac{du}{u} = 0$

65. $\int_{-1}^{3} \frac{x \, dx}{x^2+1}$

$u = x^2+1 \qquad u(-1) = (-1)^2+1 = 2$

$du = 2x \, dx \qquad u(3) = 3^2+1 = 10$

$\frac{1}{2} du = x \, dx$

$$\frac{1}{2}\int_2^{10} \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_2^{10}$$
$$= \frac{1}{2}(\ln 10 - \ln 2)$$
$$= \frac{1}{2}\ln 5$$

66. $\int_0^2 \frac{e^x dx}{3+e^x}$

$u = 3+e^x \qquad u(0) = 3+e^0 = 3+1 = 4$

$du = e^x \, dx \qquad u(2) = 3+e^2$

$$\int_4^{3+e^2} \frac{du}{u} = \ln|u|\Big|_4^{3+e^2}$$
$$= \ln(3+e^2) - \ln 4$$

67. Let $u = x^4+9$, $du = 4x^3 \; dx$.

$u(0) = 0+9 = 9, \qquad u(1) = 1+9 = 10$

(a) $\int_0^1 \frac{x^3 dx}{\sqrt{x^4+9}} = \int_9^{10} \frac{1}{4} u^{-1/2} \; du$
$$= \frac{1}{2}u^{1/2}\Big|_9^{10}$$
$$= \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9}$$
$$= \frac{1}{2}\sqrt{10} - \frac{3}{2}$$

(b) $\int \frac{x^3}{x^4+9} dx = \int \frac{1}{4} u^{-1/2} \, du$
$$= \frac{1}{2}u^{1/2} + C$$
$$= \frac{1}{2}\sqrt{x^4+9} + C$$

$$\int_0^1 \frac{x^3}{x^4+9} \; dx = \frac{1}{2}\sqrt{x^4+9}\Big|_0^1$$
$$= \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9}$$
$$= \frac{1}{2}\sqrt{10} - \frac{3}{2}$$

68. Let $u = 1 - \cos 3x$, $du = 3 \sin 3x \, dx$.

$u\left(\frac{\pi}{6}\right) = 1 - \cos\frac{\pi}{2} = 1$

$u\left(\frac{\pi}{3}\right) = 1 - \cos \pi = 2$

(a) $\int_{\pi/6}^{\pi/3} (1-\cos 3x)\sin 3x \, dx = \int_1^2 \frac{1}{3} u \, du$
$$= \frac{1}{6}u^2\Big|_1^2$$
$$= \frac{1}{6}(2)^2 - \frac{1}{6}(1)^2$$
$$= \frac{1}{2}$$

(b) $\int (1-\cos 3x)\sin 3x \; dx = \int \frac{1}{3} u \; du$
$$= \frac{1}{6}u^2 + C$$
$$= \frac{1}{6}(1-\cos 3x)^2 + C$$

$$\int_{\pi/6}^{\pi/3} (1-\cos 3x)\sin 3x \, dx$$
$$= \frac{1}{6}(1-\cos 3x)^2\Big|_{\pi/6}^{\pi/3}$$
$$= \frac{1}{6}(2)^2 - \frac{1}{6}(1)^2$$
$$= \frac{1}{2}$$

69. We show that $f'(x) = \tan x$ and $f(3) = 5$, where $f(x) = \ln\left|\frac{\cos 3}{\cos x}\right| + 5$.

$$f'(x) = \frac{d}{dx}\left(\ln\left|\frac{\cos 3}{\cos x}\right| + 5\right)$$
$$= \frac{d}{dx}\left(\ln|\cos 3| - \ln|\cos x| + 5\right)$$
$$= -\frac{d}{dx}\ln|\cos x|$$
$$= -\frac{1}{\cos x}(-\sin x)$$
$$= \tan x$$
$$f(3) = \ln\left|\frac{\cos 3}{\cos 3}\right| + 5 = \ln 1 + 5 = 5$$

70. We show that $f'(x) = \cot x$ and $f(2) = 6$, where

$$f(x) = \ln\left|\frac{\sin x}{\sin 2}\right| + 6 = \ln|\sin x| - \ln|\sin 2| + 6$$
$$f'(x) = \frac{d}{dx}\left(\ln|\sin x| - \ln|\sin 2| + 6\right)$$
$$= \frac{1}{\sin x}\cdot\cos x - 0 + 0$$
$$= \cot x$$
$$f(2) = \ln\left|\frac{\sin 2}{\sin 2}\right| + 6 = \ln 1 + 6 = 6$$

71. False; the interval of integration should change from $\left[0, \frac{\pi}{4}\right]$ to [0, 1], resulting in a different numerical answer.

72. True; use the substitution $u = f(x)$, $du = f'(x)\,dx$:

$$\int_a^b \frac{f'(x)dx}{f(x)} = \int_{f(a)}^{f(b)} \frac{du}{u}$$
$$= \ln|u|\Big|_{f(a)}^{f(b)}$$
$$= \ln|f(b)| - \ln|f(a)|$$
$$= \ln f(b) - \ln f(a)$$
$$= \ln\left(\frac{f(b)}{f(a)}\right)$$

73. D

74. E; $\int_0^2 e^{2x}dx = \frac{e^{2x}}{2}\Big|_0^2 = \frac{e^4 - 1}{2}$

75. B; $\int_3^5 f(x-a)\,dx = F(x-a)\Big|_3^5$

$$= F(5-a) - F(3-a)$$
$$= 7$$
$$\int_{3-a}^{5-a} f(x)\,dx = F(x)\Big|_{3-a}^{5-a}$$
$$= F(5-a) - F(3-a)$$
$$= 7$$

76. A; $\frac{d}{dx}\sin x = \cos x$

$$\cos\left(-\frac{\pi}{2}\right) = 0$$
$$\cos(0) = 1$$
$$\cos\left(\frac{\pi}{2}\right) = 0$$

77. **(a)** Let $u = x + 1$

$du = dx$

$$\int\sqrt{x+1}\,dx = \int u^{1/2}du$$
$$= \frac{2}{3}u^{3/2} + C$$
$$= \frac{2}{3}(x+1)^{3/2} + C$$

Alternatively,

$$\frac{d}{dx}\left(\frac{2}{3}(x+1)^{3/2} + C\right) = \sqrt{x+1}.$$

(b) By Part 1 of the Fundamental Theorem of Calculus, $\frac{dy_1}{dx} = \sqrt{x+1}$ and $\frac{dy_2}{dx} = \sqrt{x+1}$, so both are antiderivatives of $\sqrt{x+1}$.

(c) Using NINT to find the values of y_1 and y_2, we have:

x	0	1	2	3	4
y_1	0	1.219	2.797	4.667	6.787
y_2	–4.667	–3.448	–1.869	0	2.120
$y_1 - y_2$	4.667	4.667	4.667	4.667	4.667

$C = 4\frac{2}{3}$

(d) $C = y_1 - y_2$

$$= \int_0^x \sqrt{x+1}\ dx - \int_3^x \sqrt{x+1}\ dx$$
$$= \int_0^x \sqrt{x+1}\ dx + \int_x^3 \sqrt{x+1}\ dx$$
$$= \int_0^3 \sqrt{x+1}\ dx$$

78. (a) $\frac{d}{dx}[F(x)+C]$ should equal $f(x)$.

(b) The slope field should help you visualize the solution curve $y = F(x)$.

(c) The graphs of $y_1 = F(x)$ and $y_2 = \int_0^x f(t)\,dt$ should differ only by a vertical shift C.

(d) A table of values for $y_1 - y_2$ should show that $y_1 - y_2 = C$ for any value of x in the appropriate domain.

(e) The graph of f should be the same as the graph of NDER of $F(x)$.

(f) First, we need to find $F(x)$. Let $u = x^2 + 1$, $du = 2x\,dx$.

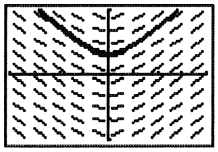
$$\int \frac{x}{\sqrt{x^2+1}}\ dx = \int \frac{1}{2} u^{-1/2}\ du$$
$$= u^{1/2}$$
$$= \sqrt{x^2+1} + C$$

Therefore, we may let $F(x) = \sqrt{x^2+1}$.

(a) $\frac{d}{dx}(\sqrt{x^2+1} + C) = \frac{1}{2\sqrt{x^2+1}}(2x)$

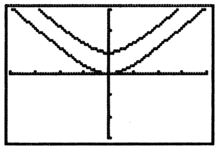
$$= \frac{x}{\sqrt{x^2+1}}$$
$$= f(x)$$

(b)



[−4, 4] by [−3, 3]

(c)

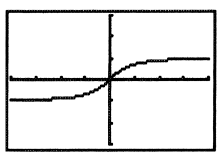


[−4, 4] by [−3, 3]

(d)

x	0	1	2	3	4
y_1	1.000	1.414	2.236	3.162	4.123
y_2	0.000	0.414	1.236	2.162	3.123
$y_1 - y_2$	1	1	1	1	1

(e)



[−4, 4] by [−3, 3]

79. **(a)** $\int 2\sin x\cos x\,dx = \int 2u\,du$

$$= u^2 + C$$

$$= \sin^2 x + C$$

(b) $\int 2\sin x\cos x\,dx = -\int 2u\,du$

$$= -u^2 + C$$

$$= -\cos^2 x + C$$

(c) Since $\sin^2 x - (-\cos^2 x) = 1,$ the two answers differ by a constant (accounted for in the constant of integration).

80. **(a)** $\int 2\sec^2 x\tan x\,dx = \int 2u\,du$

$$= u^2 + C$$

$$= \tan^2 x + C$$

(b) $\int 2\sec^2 x\tan x\,dx = \int 2u\,du$

$$= u^2 + C$$

$$= \sec^2 x + C$$

(c) Since $\sec^2 x - \tan^2 x = 1$, the two answers differ by a constant (accounted for in the constant of integration).

81. (a) $$\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{\cos u\, du}{\sqrt{1-\sin^2 u}} = \frac{\cos u\, du}{\sqrt{\cos^2 u}} = \int 1 du.$$

(Note $\cos u > 0$, so $\sqrt{\cos^2 u} = |\cos u| = \cos u$.)

(b) $$\int \frac{dx}{\sqrt{1-x^2}} = \int 1 du = u + C = \sin^{-1} x + C$$

82. (a) $$\int \frac{dx}{1+x^2} = \int \frac{\sec^2 u\, du}{1+\tan^2 u} = \int \frac{\sec^2 u\, du}{\sec^2 u} = \int 1\, du$$

(b) $$\int \frac{dx}{1+x^2} = \int 1\, du = u + C = \tan^{-1} x + C$$

83. (a) $$\int_0^{1/2} \frac{\sqrt{x}dx}{\sqrt{1-x}} = \int_{\sin^{-1}\sqrt{0}}^{\sin^{-1}\sqrt{1/2}} \frac{\sin y \cdot 2\sin y \cos y\, dy}{\sqrt{1-\sin^2 y}} = \int_0^{\pi/4} \frac{2\sin^2 y \cos y\, dy}{\cos y} = \int_0^{\pi/4} 2\sin^2 y\, dy$$

(b) $$\int_0^{1/2} \frac{\sqrt{x}dx}{\sqrt{1-x}} = \int_0^{\pi/4} 2\sin^2 y\, dy = \int_0^{\pi/4} (1-\cos 2y)dy = [y - (1/2)\sin 2y]_0^{\pi/4} = \left[\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right] - [0-0] = \frac{(\pi-2)}{4}$$

84. (a) $$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{1+x^2}} = \int_{\tan^{-1} 0}^{\tan^{-1}\sqrt{3}} \frac{\sec^2 u\, du}{\sqrt{1+\tan^2 u}} = \int_0^{\pi/3} \frac{\sec^2 u\, du}{\sec u} = \int_0^{\pi/3} \sec u\, du$$

(b) $$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{1+x^2}} = \int_0^{\pi/3} \sec u\, du = \left[\ln|\sec u + \tan u|\right]_0^{\pi/3} = \ln\left(2+\sqrt{3}\right) - \ln(1+0) = \ln\left(2+\sqrt{3}\right)$$

Section 7.3 Antidifferentiation by Parts (pp. 345–352)

Exploration 1 Choosing the Right u and dv

1. $u = 1 \qquad du = 0$

$dv = x\cos x \qquad v = \int x \cos x\, dx$

Using 1 for u is never a good idea because it places us back where we started.

2. $u = x\cos x \qquad du = \cos x - x\sin x$

$dv = dx \qquad v = \int dx = 1$

The selection of $u = x \cos x$ will place a more difficult integral into $\int v\, du$.

3. $u = \cos x \qquad du = -\sin x$

$dv = x\,dx \qquad v = \int x\,dx = x^2$

The selection of $dv = x\,dx$ will place a more difficult integral into $\int v\, du$.

4. $u = x$ and $dv = \cos x\, dx$ are good choices because the integral is simplified.

Quick Review 7.3

1. $$\frac{dy}{dx} = (x^3)(\cos 2x)(2) + (\sin 2x)(3x^2) = 2x^3 \cos 2x + 3x^2 \sin 2x$$

2. $$\frac{dy}{dx} = (e^{2x})\left(\frac{3}{3x+1}\right) + \ln(3x+1)(2e^{2x}) = \frac{3e^{2x}}{3x+1} + 2e^{2x}\ln(3x+1)$$

3. $\dfrac{dy}{dx}=\dfrac{1}{1+(2x)^2}\cdot 2=\dfrac{2}{1+4x^2}$

4. $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{1-(x+3)^2}}$

5.
$$y=\tan^{-1}3x$$
$$\tan y=3x$$
$$x=\frac{1}{3}\tan y$$

6.
$$y=\cos^{-1}(x+1)$$
$$\cos y=x+1$$
$$x=\cos y-1$$

7.
$$\int_0^1 \sin \pi x\,dx=-\frac{1}{\pi}\cos \pi x\Big|_0^1$$
$$=-\frac{1}{\pi}\cos\pi+\frac{1}{\pi}\cos 0$$
$$=-\frac{1}{\pi}(-1)+\frac{1}{\pi}$$
$$=\frac{2}{\pi}$$

8. $\dfrac{dy}{dx}=e^{2x}$

$dy=e^{2x}dx$

Integrate both sides.

$\int dy=\int e^{2x}dx$

$y=\dfrac{1}{2}e^{2x}+C$

9. $\dfrac{dy}{dx}=x+\sin x$

$dy=(x+\sin x)dx$

Integrate both sides.

$\int dy=\int (x+\sin x)dx$

$y=\dfrac{1}{2}x^2-\cos x+C$

$y(0)=-1+C=2,\ C=3$

$y=\dfrac{1}{2}x^2-\cos x+3$

10.
$$\frac{d}{dx}\left(\frac{1}{2}e^x(\sin x-\cos x)\right)$$
$$=\frac{1}{2}e^x(\cos x+\sin x)+(\sin x-\cos x)\frac{1}{2}e^x$$
$$=\frac{1}{2}e^x\cos x+\frac{1}{2}e^x\sin x+\frac{1}{2}e^x\sin x-\frac{1}{2}e^x\cos x$$
$$=e^x\sin x$$

Section 7.3 Exercises

1. $\int x\sin x\,dx$

$dv=\sin x\,dx \qquad v=\int \sin x\,dx=-\cos x$

$u=x \qquad du=dx$

$-x\cos x-\int -\cos x\,dx=-x\cos x+\sin x+C$

2. $\int xe^x dx$

$dv=e^x dx \qquad v=\int e^x dx=e^x$

$u=x \qquad du=dx$

$xe^x-\int e^x dx=xe^x-e^x+C$

3. $\int 3\,t\,e^{2t}dt$

$dv=e^{2t}dt \qquad v=\int e^{2t}dt=\dfrac{e^{2t}}{2}$

$u=3t \qquad du=3\,dt$

$3t\,\dfrac{e^{2t}}{2}-\int 3\,\dfrac{e^{2t}}{2}dt=\dfrac{3}{2}te^{2t}-\dfrac{3}{4}e^{2t}+C$

4. $\int 2\,t\cos(3t)\,dt$

$dv=\cos 3t\,dt \quad v=\int \cos(3t)\,dt=\dfrac{\sin 3t}{3}$

$u=2t \qquad du=2\,dt$

$2t\,\dfrac{\sin 3t}{3}-\int 2\dfrac{\sin 3t}{3}dt$

$=\dfrac{2}{3}t\sin 3t-\dfrac{2}{9}\cos(3t)+C$

5. $\int x^2\cos x\,dx$

$dv=\cos x\,dx \qquad v=\int \cos x\,dx=\sin x$

$u=x^2 \qquad du=2x\,dx$

$x^2\sin x-\int 2x\sin x\,dx$

$dv=\sin x\,dx \qquad v=\int \sin x\,dx=-\cos x$

$u=2x \qquad du=2dx$

$x^2\sin x+2x\cos x-\int 2\cos x dx$

$=x^2\sin x+2x\cos x-2\sin x+C$

6. $\int x^2 e^{-x} dx$

$dv = e^{-x} dx \quad v = \int e^{-x} dx = -e^{-x}$

$u = x^2 \quad du = 2x\, dx$

$-x^2 e^{-x} - \int -2x\, e^{-x} dx = -x^2 e^{-x} + \int 2xe^{-x} + C$

$dv = e^{-x} \quad v = \int e^{-x} dx = -e^{-x}$

$u = 2x \quad du = 2\, dx$

$-x^2 e^{-x} - 2xe^{-x} - \int -2e^{-x} dx$

$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + C$

7. $\int 3x^2\, e^{2x} dx$

$dv = e^{2x} dx \quad v = \int e^{2x} dx = \frac{e^{2x}}{2}$

$u = 3x^2 \quad du = 6x$

$3x^2 \frac{e^{2x}}{2} - \int 6x \frac{e^{2x}}{2} dx = \frac{3}{2} x^2 e^{2x} - \int 3x\, e^{2x} dx$

$dv = e^{2x} \quad v = \int e^{2x} dx = \frac{e^{2x}}{2}$

$u = 3x \quad du = 3dx$

$\frac{3}{2} x^2 e^{2x} - \left[\frac{3}{2} xe^{2x} - \int 3 \frac{e^{2x}}{2} dx \right]$

$= \frac{3}{2} x^2 e^{2x} - \frac{3}{2} xe^{2x} + \frac{3}{4} e^{2x} + C$

8. $\int x^2 \cos\left(\frac{x}{2}\right) dx$

$dv = \cos\left(\frac{x}{2}\right) dx \quad v = \int \cos\left(\frac{x}{2}\right) dx = 2\sin\left(\frac{x}{2}\right)$

$u = x^2 \quad du = 2x\, dx$

$2x^2 \sin\left(\frac{x}{2}\right) - \int 4x \sin\left(\frac{x}{2}\right) dx$

$dv = \sin\left(\frac{x}{2}\right) \quad v = \int \sin\left(\frac{x}{2}\right) dx = -2\cos\left(\frac{x}{2}\right)$

$u = 4x \quad du = 4dx$

$2x^2 \sin\left(\frac{x}{2}\right) + 8x\cos\left(\frac{x}{2}\right) - \int 8\cos\left(\frac{x}{2}\right) dx$

$= 2x^2 \sin\left(\frac{x}{2}\right) + 8x\cos\left(\frac{x}{2}\right) - 16\sin\left(\frac{x}{2}\right) + C$

9. $\int y \ln y\, dy$

$dv = y\, dy \quad v = \int y\, dy = \frac{y^2}{2}$

$u = \ln y \quad du = \frac{1}{y} dy$

$\frac{1}{2} y^2 \ln y - \int \frac{y^2}{2} \frac{1}{y} dy = \frac{1}{2} y^2 \ln y - \frac{y^2}{4} + C$

10. $\int t^2 \ln t\, dt$

$dv = t^2\, dt \quad v = \int t^2 dt = \frac{t^3}{3}$

$u = \ln t \quad du = \frac{1}{t} dt$

$\frac{1}{3} t^3 \ln t - \int \frac{t^3}{3} \frac{1}{t} dt = \frac{1}{3} t^3 \ln t - \frac{t^3}{9} + C$

11. $\int dy = \int \big((x+2)\sin x\big) dx$

$dv = \sin x\, dx \quad v = \int \sin x\, dx = -\cos x$

$u = x + 2 \quad du = dx$

$-(x+2)\cos x - \int (-\cos x)\, dx$

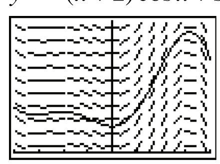
$= -(x+2)\cos x + \sin x + C$

$2 = -(0+2)\cos(0) + \sin(0) + C$

$2 = -2 + C$

$C = 4$

$y = -(x+2)\cos x + \sin x + 4$



[–4, 4] by [0, 10]

12. $\int dy = \int 2xe^{-x} dx$

$dv = e^{-x} dx \quad v = \int e^{-x} dx = -e^{-x}$

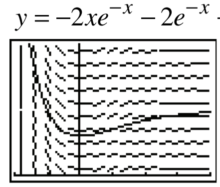
$u = 2x \quad du = 2\, dx$

$-2xe^{-x} - \int -2e^{-x} dx = -2xe^{-x} - 2e^{-x} + C$

$3 = -2(0)\, e^{(-0)} - 2e^{(-0)} + C$

$5 = C$

$y = -2xe^{-x} - 2e^{-x} + 5$



[–2, 4] by [0, 10]

13. $\int du = \int x \sec^2 x\, dx$

$dv = \sec^2 x\, dx \qquad v = \int \sec^2 x\, dx = \tan x$

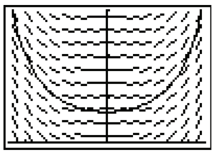
$w = x \qquad dw = dx$

$x \tan x - \int \tan x\, dx = x \tan x + \ln|\cos x| + C$

$1 = 0 \tan(0) + \ln|\cos(0)| + C$

$C = 1$

$u = x \tan(x) + \ln|\cos(x)| + 1$



[−1.2, 1.2] by [0, 3]

14. $\int dz = x^3 \ln x\, dx$

$dv = x^3 dx \qquad v = \int x^3 dx = \frac{x^4}{4}$

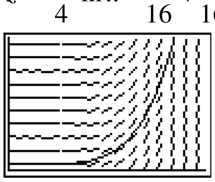
$u = \ln x \qquad du = \frac{1}{x} dx$

$\frac{x^4}{4} \ln x - \int \frac{x^4}{4} \frac{1}{x} dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + C$

$5 = \frac{(1)^4}{4} \ln(1) - \frac{(1)^4}{16} + C$

$C = \frac{81}{16}$

$z = \frac{x^4}{4} \ln x - \frac{x^4}{16} + \frac{81}{16}$



[0, 5] by [0, 100]

15. $\int dy = \int x\sqrt{x-1}\, dx$

$dv = (x-1)^{1/2} dx \qquad v = \int (x-1)^{1/2} dx = \frac{2}{3}(x-1)^{3/2}$

$u = x \qquad du = dx$

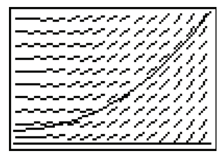
$\frac{2}{3} x (x-1)^{3/2} - \int \frac{2}{3}(x-1)^{3/2} dx$

$= \frac{2}{3} x (x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + C$

$2 = \frac{2}{3}(1)(1-1)^{3/2} - \frac{4}{15}(1-1)^{5/2} + C$

$C = 2$

$y = \frac{2}{3} x (x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + 2$



[1, 5] by [0, 20]

16. $\int dy = \int 2x\sqrt{x+2}\, dx$

$dv = (x+2)^{1/2} dx \qquad v = \int (x+2)^{1/2} dx = \frac{2}{3}(x+2)^{3/2}$

$u = 2x \qquad du = 2dx$

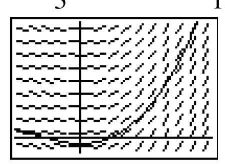
$\frac{4}{3} x (x+2)^{3/2} - \int \frac{4}{3}(x+2)^{3/2} dx$

$= \frac{4}{3} x (x+2)^{3/2} - \frac{8}{15}(x+2)^{5/2} + C$

$0 = \frac{4}{3}(-1)(-1+2)^{3/2} - \frac{8}{15}(-1+2)^{5/2} + C$

$C = \frac{28}{15}$

$y = \frac{4}{3} x (x+2)^{3/2} - \frac{8}{15}(x+2)^{5/2} + \frac{28}{15}$



[−2, 4] by [−3, 25]

17. $\int e^x \sin x\,dx$

$dv = e^x\,dx \quad v = \int e^x\,dx = e^x$

$u = \sin x \quad du = \cos x\,dx$

$e^x \sin x - \int e^x \cos x\,dx$

$dv = e^x\,dx \quad v = \int e^x\,dx = e^x$

$u = \cos x \quad du = -\sin x\,dx$

$\int e^x \sin x\,dx = e^x \sin x - \left[e^x \cos x - \int -e^x \sin x\,dx\right]$

$\int e^x \sin x\,dx = \dfrac{e^x}{2}(\sin x - \cos x) + C$

18. $\int e^{-x} \cos\ x\,dx$

$dv = \cos x\,dx \quad v = \int \cos x\,dx = \sin x$

$u = e^{-x} \quad du = -e^{-x}\,dx$

$e^{-x} \sin x - \int -e^{-x} \sin x\,dx$

$dv = \sin x\,dx \quad v = \int \sin x\,dx = -\cos x$

$u = e^{-x} \quad du = -e^{-x}\,dx$

$\int e^{-x} \cos x\,dx = e^{-x} \sin x - \left[e^{-x} \cos x - \int -e^{-x} \cos x\,dx\right]$

$\int e^{-x} \cos x\,dx = \dfrac{e^{-x}}{2}(\sin x - \cos x) + C$

19. $\int e^x \cos\ 2x\,dx$

$dv = \cos 2x\,dx \quad v = \int \cos 2x\,dx = \dfrac{1}{2}\sin 2x$

$u = e^x \quad du = e^x dx$

$\dfrac{1}{2}e^x \sin 2x - \int \dfrac{1}{2}\sin 2x\ e^x dx$

$dv = \dfrac{1}{2}\sin 2x\,dx \quad v = \int \dfrac{1}{2}\sin 2x\,dx = -\dfrac{1}{4}\cos 2x$

$u = e^x \quad du = e^x dx$

$\int e^x \cos 2x\,dx = \dfrac{1}{2}e^x \sin 2x - \left[-\dfrac{1}{4}e^x \cos 2x - \int -\dfrac{1}{4}\cos 2x\ e^x dx\right]$

$\int e^x \cos 2x\,dx = \dfrac{1}{2}e^x \sin 2x + \dfrac{1}{4}e^x \cos 2x - \dfrac{1}{4}\int e^x \cos 2x\,dx$

$\dfrac{5}{4}\int e^x \cos 2x\,dx = \dfrac{1}{4}e^x(2\sin 2x + \cos 2x)$

$\int e^x \cos 2x\,dx = \dfrac{e^x}{5}(2\sin 2x + \cos 2x) + C$

20. $\int e^{-x} \sin 2x \, dx$

$$dv = \sin 2x \, dx \qquad v = \int \sin 2x \, dx = -\frac{1}{2}\cos 2x$$
$$u = e^{-x} \qquad du = -e^{-x} dx$$
$$-\frac{1}{2}e^{-x}\cos 2x - \int \frac{1}{2}\cos 2x \, e^{-x} dx$$
$$dv = \frac{1}{2}\cos 2x \, dx \quad v = \int \frac{1}{2}\cos 2x \, dx = \frac{1}{4}\sin 2x$$
$$u = e^{-x} \qquad du = -e^{-x} dx$$
$$\int e^{-x}\sin 2x \, dx = -\frac{1}{2}e^{-x}\cos 2x - \left[\frac{1}{4}e^{-x}\sin 2x - \int -\frac{1}{4}e^{-x}\sin 2x \, dx\right]$$
$$\int e^{-x}\sin 2x \, dx = -\frac{1}{2}e^{-x}\cos 2x - \frac{1}{4}e^{-x}\sin 2x - \frac{1}{4}\int e^{-x}\sin 2x \, dx$$
$$\frac{5}{4}\int e^{-x}\sin 2x \, dx = -\frac{1}{4}e^{-x}(2\cos 2x + \sin 2x)$$
$$\int e^{-x}\sin 2x \, dx = -\frac{e^{-x}}{5}(2\cos 2x + \sin 2x) + C$$

21. Use tabular integration with $f(x) = x^4$ and $g(x) = e^{-x}$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^4	(+)	e^{-x}
$4x^3$	(−)	$-e^{-x}$
$12x^2$	(+)	e^{-x}
$24x$	(−)	$-e^{-x}$
24	(+)	e^{-x}
0		$-e^{-x}$

$$\int x^4 e^{-x} \, dx = -x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24xe^{-x} - 24e^{-x} + C$$
$$= -(x^4 + 4x^3 + 12x^2 + 24x + 24)e^{-x} + C$$

22. Use tabular integration with $f(x) = x^2 - 5x$ and $g(x) = e^x$

$f(x)$ and its derivatives		$g(x)$ and its integrals
$x^2 - 5x$	(+)	e^x
$2x - 5$	(−)	e^x
2	(+)	e^x
0		e^x

$$\int (x^2 - 5x)e^x \, dx = (x^2 - 5x)e^x - (2x - 5)e^x + 2e^x$$
$$= (x^2 - 7x + 7)e^x + C$$

23. Use tabular integration with $f(x)=x^3$ and $g(x)=e^{-2x}$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	(+)	e^{-2x}
$3x^2$	(−)	$-\frac{1}{2}e^{-2x}$
$6x$	(+)	$\frac{1}{4}e^{-2x}$
6	(−)	$-\frac{1}{8}e^{-2x}$
0		$\frac{1}{16}e^{-2x}$

$$\int x^3e^{-2x}\,dx=-\frac{1}{2}x^3e^{-2x}-\frac{3}{4}x^2e^{-2x}-\frac{3}{4}xe^{-2x}-\frac{3}{8}e^{-2x}+C$$
$$=-\left(\frac{x^3}{2}+\frac{3x^2}{4}+\frac{3x}{4}+\frac{3}{8}\right)e^{-2x}+C$$

24. Use tabular integration with $f(x)=x^3$ and $g(x)=\cos 2x$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	(+)	$\cos 2x$
$3x^2$	(−)	$\frac{1}{2}\sin 2x$
$6x$	(+)	$-\frac{1}{4}\cos 2x$
6	(−)	$-\frac{1}{8}\sin 2x$
0		$\frac{1}{16}\cos 2x$

$$\frac{x^3}{2}\sin 2x+\frac{3x^2}{4}\cos 2x-\frac{3x}{4}\sin 2x-\frac{3}{8}\cos 2x+C$$

25. Use tabular integration with $f(x)=x^2$ and $g(x)=\sin 2x$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	(+)	$\sin 2x$
$2x$	(−)	$-\frac{1}{2}\cos 2x$
2	(+)	$-\frac{1}{4}\sin 2x$
0		$\frac{1}{8}\cos 2x$

$$\int x^2\sin 2x\,dx=-\frac{1}{2}x^2\cos 2x+\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x+C$$
$$=\left(\frac{1-2x^2}{4}\right)\cos\ 2x+\frac{x}{2}\sin 2x+C$$

$$\int_0^{\pi/2} x^2 \sin 2x\,dx = \left[\left(\frac{1-2x^2}{4}\right)\cos\ 2x + \frac{x}{2}\sin\ 2x\right]_0^{\pi/2}$$
$$= \left(\frac{1-2\left(\frac{\pi}{2}\right)^2}{4}\right)(-1)+0-\left(\frac{1}{4}\right)(1)-0$$
$$= \frac{\pi^2}{8}-\frac{1}{2}$$

26. Use tabular integration with $f(x)=x^3$ and $g(x)=\cos 2x$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	(+)	$\cos 2x$
$3x^2$	(−)	$\frac{1}{2}\sin 2x$
$6x$	(+)	$-\frac{1}{4}\cos 2x$
6	(−)	$-\frac{1}{8}\sin 2x$
0		$\frac{1}{16}\cos 2x$

$$\int x^3 \cos 2x\,dx = \frac{1}{2}x^3\ \sin\ 2x + \frac{3}{4}x^2\ \cos\ 2x - \frac{3}{4}x$$
$$\sin\ 2x - \frac{3}{8}\cos\ 2x = \left(\frac{x^3}{2}-\frac{3x}{4}\right)\sin\ 2x + \left(\frac{3x^2}{4}-\frac{3}{8}\right)\cos\ 2x + C$$

$$\int_0^{\pi/2} x^3 \cos 2x\,dx = \left[\left(\frac{x^3}{2}-\frac{3x}{4}\right)\sin\ 2x + \left(\frac{3x^2}{4}-\frac{3}{8}\right)\cos\ 2x\right]_0^{\pi/2}$$
$$= 0+\left(\frac{3\pi^2}{16}-\frac{3}{8}\right)(-1)-0-\left(-\frac{3}{8}\right)(1)$$
$$= \frac{3}{4}-\frac{3\pi^2}{16}$$

27. Let $u = e^{2x}$ $\qquad dv = \cos 3x\ dx$

$du = 2e^{2x}\ dx \qquad v = \frac{1}{3}\sin 3x$

$$\int e^{2x}\cos 3x\,dx = (e^{2x})\left(\frac{1}{3}\sin 3x\right) - \int\left(\frac{1}{3}\sin 3x\right)(2e^{2x}\,dx)$$
$$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx$$

Let $u = e^{2x}$ $\quad dv = \sin 3x\, dx$

$du = 2e^{2x}\, dx$ $\quad v = -\frac{1}{3}\cos 3x$

$$\int e^{2x} \cos 3x\, dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[(e^{2x})\left(-\frac{1}{3}\cos 3x\right) - \int\left(-\frac{1}{3}\cos 3x\right)(2e^{2x}\, dx)\right]$$

$$= \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x) - \frac{4}{9}\int e^{2x}\cos 3x\, dx$$

$$\frac{13}{9}\int e^{2x}\cos 3x\, dx = \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x)$$

$$\int e^{2x}\cos 3x\, dx = \frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)$$

$$\int_{-2}^{3} e^{2x}\cos 3x\, dx = \left[\frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)\right]_{-2}^{3}$$

$$= \frac{1}{13}[e^6(3\sin 9 + 2\cos 9) - e^{-4}(3\sin(-6) + 2\cos(-6)]$$

$$= \frac{1}{13}[e^6(2\cos 9 + 3\sin 9) - e^{-4}(2\cos 6 - 3\sin 6)]$$

28. Let $u = e^{-2x}$ $\quad dv = \sin 2x\, dx$

$du = -2e^{-2x}\, dx$ $\quad v = -\frac{1}{2}\cos 2x$

$$\int e^{-2x}\sin 2x\, dx = (e^{-2x})\left(-\frac{1}{2}\cos 2x\right) - \int\left(-\frac{1}{2}\cos 2x\right)(-2e^{-2x}\, dx)$$

$$= -\frac{1}{2}e^{-2x}\cos 2x - \int e^{-2x}\cos 2x\, dx$$

Let $u = e^{-2x}$ $\quad dv = \cos 2x\, dx$

$du = -2e^{-2x}dx$ $\quad v = \frac{1}{2}\sin 2x$

$$\int e^{-2x}\sin 2x\, dx = -\frac{1}{2}e^{-2x}\cos 2x - \left[(e^{-2x})\left(\frac{1}{2}\sin 2x\right) - \int\left(\frac{1}{2}\sin 2x\right)(-2e^{-2x}\, dx)\right]$$

$$= -\frac{1}{2}e^{-2x}(\cos 2x + \sin 2x) - \int e^{-2x}\sin 2x\, dx$$

$$2\int e^{-2x}\sin 2x\, dx = -\frac{1}{2}e^{-2x}(\cos 2x + \sin 2x) + C$$

$$\int e^{-2x}\sin 2x\, dx = -\frac{e^{-2x}}{4}(\cos 2x + \sin 2x) + C$$

$$\int_{-3}^{2} e^{-2x}\sin 2x\, dx = \left[-\frac{e^{-2x}}{4}(\cos 2x + \sin 2x)\right]_{-3}^{2}$$

$$= -\frac{e^{-4}}{4}(\cos 4 + \sin 4) + \frac{e^6}{4}[\cos(-6) + \sin(-6)]$$

$$= -\frac{e^{-4}}{4}(\cos 4 + \sin 4) + \frac{e^6}{4}(\cos 6 - \sin 6)$$

29. $y = \int x^2 e^{4x}\,dx$

Let $u = x^2 \qquad dv = e^{4x}dx$

$du = 2x\,dx \qquad v = \frac{1}{4}e^{4x}$

$$y = (x^2)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)(2x\,dx)$$
$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\int xe^{4x}dx$$

Let $u = x \qquad dv = e^{4x}\,dx$

$du = dx \qquad v = \frac{1}{4}e^{4x}$

$$y = \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left[(x)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)dx\right]$$
$$y = \frac{1}{4}x^2\mathrm{e}^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C$$
$$y = \left(\frac{x^2}{4} - \frac{x}{8} + \frac{1}{32}\right)e^{4x} + C$$

30. $y = \int x^2 \ln x\,dx$

Let $u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{3}x^3$

$$y = (\ln x)\left(\frac{1}{3}x^3\right) - \int\left(\frac{1}{3}x^3\right)\left(\frac{1}{x}dx\right)$$
$$y = \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2dx$$
$$y = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$$

31. $y = \int \theta \sec^{-1}\theta\,d\theta$

Let $u = \sec^{-1}\theta \qquad dv = \theta\,d\theta$

$du = \frac{1}{\theta\sqrt{\theta^2-1}}du \qquad v = \frac{1}{2}\theta^2$

Note that we are told $\theta > 1$, so no absolute value is needed in the expression for du.

$$y = (\sec^{-1}\theta)\left(\frac{1}{2}\theta^2\right) - \int\left(\frac{1}{2}\theta^2\right)\left(\frac{1}{\theta\sqrt{\theta^2-1}}d\theta\right)$$
$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{2}\int\frac{\theta\,d\theta}{\sqrt{\theta^2-1}}$$

Let $w = \theta^2 - 1,\ dw = 2\theta\,d\theta$

$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{4}\int w^{-1/2}dw$$
$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{2}w^{1/2} + C$$
$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{2}\sqrt{\theta^2-1} + C$$

32. $y = \int \theta \sec\theta \tan\theta\,d\theta$

Let $u = \theta \qquad dv = \sec\theta\tan\theta\,d\theta$

$du = d\theta \qquad v = \sec\theta$

$$y = \theta\sec\theta - \int\sec\theta\,d\theta$$
$$y = \theta\sec\theta - \ln|\sec\theta + \tan\theta| + C$$

Note : In the last step, we used the result of Exercise 45 in Section 7.2.

33. Let $u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$$\int x\sin x\,dx = -x\cos x + \int\cos x\,dx$$
$$= -x\cos x + \sin x + C$$

(a) $\int_0^{\pi}|x\sin x|\,dx = \int_0^{\pi}x\sin x\,dx$

$$= [-x\cos x + \sin x]_0^{\pi}$$
$$= -\pi(-1) + 0 + 0(1) - 0$$
$$= \pi$$

(b) $\int_{\pi}^{2\pi}|x\sin x|\,dx = -\int_{\pi}^{2\pi}x\sin x\,dx$

$$= [x\cos x - \sin x]_{\pi}^{2\pi}$$
$$= 2\pi(1) - 0 - \pi(-1) + 0$$
$$= 3\pi$$

(c) $\int_0^{2\pi}|x\sin x|\,dx$

$$= \int_0^{\pi}|x\sin x|\,dx + \int_{\pi}^{2\pi}|x\sin x|\,dx$$
$$= \pi + 3\pi = 4\pi$$

34. We begin by evaluating $\int (x^2+x+1)e^{-x}\,dx$.

Let $u = x^2+x+1 \qquad dv = e^{-x}\,dx$

$du = (2x+1)\,dx \qquad v = -e^{-x}$

$\int (x^2+x+1)e^{-x}\,dx$

$= -(x^2+x+1)e^{-x} + \int (2x+1)e^{-x}\,dx$

Let $u = 2x+1 \qquad dv = e^{-x}\,dx$

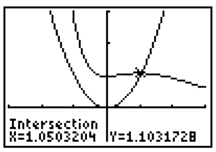
$du = 2\,dx \qquad v = -e^{-x}$

$\int (x^2+x+1)e^{-x}\,dx$

$= -(x^2+x+1)e^{-x} - (2x+1)e^{-x} + \int 2e^{-x}\,dx$

$= -(x^2+x+1)e^{-x} - (2x+1)e^{-x} - 2e^{-x} + C$

$= -(x^2+3x+4)e^{-x} + C$



[–3, 3] by [–3, 3]

The graph shows that the two curves intersect at $x = k$, where $k \approx 1.050$. The area we seek is

$\int_0^k (x^2+x+1)e^{-x} - \int_0^k x^2\,dx$

$= \left[-(x^2+3x+4)e^{-x}\right]_0^k - \left[\frac{1}{3}x^3\right]_0^k$

$\approx (-2.888+4) - (0.386-0)$

≈ 0.726

35. First, we evaluate $\int e^{-t}\cos t\,dt$.

Let $u = e^{-t} \qquad dv = \cos t\,dt$

$du = -e^{-t}\,dt \qquad v = \sin t$

$\int e^{-t}\cos t\,dt = e^{-t}\sin t + \int \sin t\, e^{-t}\,dt$

Let $u = e^{-t} \qquad dv = \sin t\,dt$

$du = -e^{-t}\,dt \qquad v = -\cos t$

$\int e^{-t}\cos t\,dt = e^{-t}\sin t - e^{-t}\cos t - \int e^{-t}\cos t\,dt$

$2\int e^{-t}\cos t\,dt = e^{-t}(\sin t - \cos t) + C$

$\int e^{-t}\cos t\,dt = \frac{1}{2}e^{-t}(\sin t - \cos t) + C$

Now we find the average value of

$y = 2e^{-t}\cos t$ for $0 \le t \le 2\pi$.

$$\begin{aligned}\text{Average value} &= \frac{1}{2\pi}\int_0^{2\pi} 2e^{-t}\cos t\,dt\\ &= \frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t\,dt\\ &= \frac{1}{2\pi}e^{-t}(\sin t - \cos t)\Big|_0^{2\pi}\\ &= \frac{1}{2\pi}[e^{-2\pi}(-1) - e^0(-1)]\\ &= \frac{1-e^{-2\pi}}{2\pi} \approx 0.159\end{aligned}$$

36. True; use parts, letting $u = x$, $dv = g(x)dx$, and $v = f(x)$.

37. True; use parts, letting $u = x^2$, $dv = g(x)dx$, and $v = f(x)$.

38. B; $\int x^2\cos x\,dx$

$= x^2\sin x + 2x\cos x - 2\sin x + C$

See problem 5.

$\int 2x\sin x\,dx = -2x\cos x + 2\sin x + C$

See problem 1.

$h(x) = x^2\sin x + C$

39. B; $\int x\sin(5x)\,dx$

$dv = \sin(5x)dx \qquad v = \int \sin(5x)\,dx$

$= -\frac{1}{5}\cos 5x$

$u = x \qquad du = dx$

$-\frac{1}{5}x\cos(5x) - \int -\frac{1}{5}\cos(5x)\,dx$

$= -\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C$

40. C; $\int x\csc^2 x\,dx$

$dv = \csc^2 x\,dx \qquad v = \int \csc^2 x\,dx = -\cot x$

$u = x \qquad du = dx$

$-x\cot x - \int -\cot x\,dx$

$= -x\cot x + \ln|\sin x| + C$

41. C; $\int dy = \int 4x \ln x \, dx$

$dv = 4x\,dx \qquad v = \int 4x\,dx = 2x^2$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$2x^2 \ln x - \int 2x^2 \frac{1}{x}\,dx = 2x^2 \ln x - x^2 + C$

42. **(a)** Let $u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$\int xe^x\,dx = xe^x - \int e^x\,dx$$
$$= xe^x - e^x + C$$
$$= (x-1)e^x + C$$

(b) Using the result from part (a):

Let $u = x^2 \qquad dv = e^x\,dx$

$du = 2x\,dx \qquad v = e^x$

$$\int x^2 e^x\,dx = x^2 e^x - \int 2xe^x\,dx$$
$$= x^2 e^x - 2(x-1)e^x + C$$
$$= (x^2 - 2x + 2)e^x + C$$

(c) Using the result from part (b):

Let $u = x^3 \qquad dv = e^x\,dx$

$du = 3x^2 dx \qquad v = e^x$

$$\int x^3 e^x\,dx = x^3 e^x - \int 3x^2 e^x\,dx$$
$$= x^3 e^x - 3(x^2 - 2x + 2)e^x + C$$
$$= (x^3 - 3x^2 + 6x - 6)e^x + C$$

(d) $\left[x^n - \frac{d}{dx}x^n + \frac{d}{dx^2}x^n - \cdots + (-1)^n \frac{d^n}{dx^n}x^n\right]e^x + C$ or

$\left[x^n - nx^{n-1} + n(n-1)x^{n-2} - \cdots + (-1)^{n-1}(n)!x + (-1)^n(n!)\right]e^x + C$

(e) Use mathematical induction or argue based on tabular integration. Alternately, show that the derivative of the answer to part (d) is $x^n e^x$:

$$\frac{d}{dx}\left[\left(x^n - nx^{n-1} + n(n-1)x^{n-2} - \cdots + (-1)^{n-1}(n!)x + (-1)^n n!\right)e^x + C\right]$$
$$= [x^n - nx^{n-1} + n(n-1)x^{n-2} - \cdots + (-1)^{n-1}(n!)x + (-1)^n n!]e^x$$
$$+ e^x \frac{d}{dx}\left[x^n - nx^{n-1} + n(n-1)n^{n-2} - \cdots \qquad + (-1)^{n-1}(n!)x + (-1)^n n!\right]$$
$$= [x^n - nx^{n-1} + n(n-1)x^{n-2} - \cdots + (-1)^{n-1} + (n!)x + (-1)^n n!]e^x$$
$$+\left[nx^{n-1} - n(n-1)x^{n-2} + n(n-1)(n-2)x^{n-3} - \cdots + (-1)^{n-1}n!\right]e^x$$
$$= x^n e^x$$